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THERMAL AND STRESSED STATES OF A TUBE IN TRANSPORTATION OF LIQUID SODIUM

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Using the method of "superelements," the thermoelasticity problem is solved for a hollow cylinder with liquid sodium flowing inside the cylinder. The problem for each superelement is reduced to a system of difference equations solvable by the iteration method. The influence of the swelling of the crystal lattice of steel on the stressed-strained state is taken into account in the case where carbon penetrates into the lattice.

Transfer of carbon by a sodium coolant in heat-exchange installations leads to the saturation of structural elements with it. Under the action of carbon on the structure of steel, its chemical composition and physicomechanical and thermophysical characteristics change with the depth of saturation, resulting in the need for evaluating the stressed-strained state of a member in the context of the mechanics of inhomogeneous structures.

To describe the process of diffusion of carbon into a wall of a tube inside which liquid sodium flows, one must primarily know the temperature distribution in the tube at a known temperature of the coolant at the inlet.

Let us consider the solution of the problem concerning the distribution of temperatures over the tube length in a wall $T(z, r)$ and in a liquid metal $\theta(z, r)$ which flows in this tube. The temperature of the liquid sodium at the inlet to the tube is constant over the cross section. The temperature at the outer boundaries of the tube is (Fig. 1)

$$T = T_2(z) \quad \text{for} \quad r = R_2, \quad 0 \le z \le L; \quad T = T_3(r) \quad \text{for} \quad z = 0, \quad R_1 \le r \le R_2;$$

$$T = T_4(r) \quad \text{for} \quad z = L, \quad R_1 \le r \le R_2. \tag{1}$$

The temperature $T_4(R_1)$ is equal to the sodium temperature to be determined at the outlet from the tube.

The inner surface of the tube is exposed to carbon. The flow of the coolant is considered to be stabilized hydrodynamically with a Poiseuille longitudinal velocity profile, i.e., the transverse velocity component is $V_r = 0$, whereas the longitudinal velocity component is $V_z = V_z(r) = V_0(1 - r^2/R_1^2)$ and it remains constant over the tube length.

It is suggested that the thermophysical properties of the coolant are temperature-independent, while the energy dissipation due to viscous friction and the work of pressure forces are negligibly small.

The thermophysical properties of the tube material are considered to be dependent on the temperature and concentration of the carbon diffusing from the surface.

Penetration of the carbon into the tube wall is described by the differential diffusion equation for the concentration c of the diffusing substance ($0 \le c \le 1$, 1 wt.%)

$$\text{div}\,(D \,\text{grad}\, c) = \frac{\partial c}{\partial t} \tag{2}$$

with the following boundary and initial conditions:

$$c = 0 \quad \text{for} \quad R_1 \le r \le R_2, \quad t = 0; \quad c = 0 \quad \text{for} \quad r = R_2, \quad c = c(t, z) \quad \text{for} \quad r = R_1, \quad t \ge 0. \tag{3}$$

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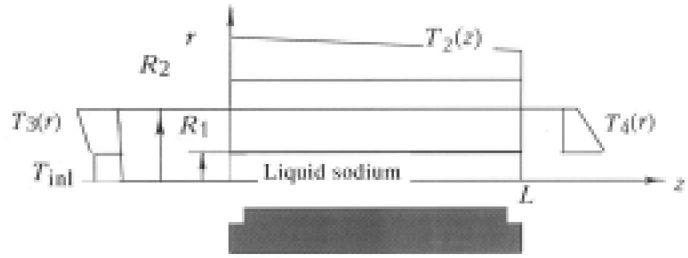

Fig. 1. Boundary conditions.

Here $D = D_0 \exp(-Q/(RT))$. The source of carbon saturation is of medium power; therefore, in conformity with the law [1] $c(t, z) = B \dfrac{k(T)+1}{k(T)} \exp\left(-\dfrac{Q}{RT}\right) t^n$, the concentration of the carbon reaches 1 wt.% and then remains constant, i.e., the inverse kinetics is not yet taken into account.

Since the rate of diffusion of the carbon into the tube wall is considerably lower than the velocity of propagation of heat, its thermal regime will be considered to be stationary. Then the temperature field in the tube wall satisfies the differential equation

$$\mathrm{div}\,(\lambda_{st}\,\mathrm{grad}\,T) = 0 \tag{4}$$

and boundary conditions (1). Here $\lambda_{st} = \lambda_{st}(c, T)$.

The steady-state energy equation describing the temperature distribution in the coolant flow at Peclet numbers $\mathrm{Pe} = V_0 R_1 / a >> 1$ (where a is the thermal diffusivity), when the heat transfer along the tube by heat conduction can be neglected compared to the convective transfer, has the form

$$c_p \rho V_z \frac{\partial \theta}{\partial z} = \frac{\lambda_{sod}}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \theta}{\partial r}\right). \tag{5}$$

The boundary conditions for Eq. (5) are as follows:

$$\left.\frac{\partial \theta}{\partial r}\right|_{r=0} = 0 \quad \text{(symmetry condition on the tube axis)}, \tag{6}$$

$$\theta\,|_{z=0} = T_{inl}, \tag{6$'$}$$

$$\lambda_{st} \left.\frac{\partial T}{\partial r}\right|_{r=R_1} = \lambda_{sod} \left.\frac{\partial \theta}{\partial r}\right|_{r=R_1}, \quad T\,|_{r=R_1} = \theta\,|_{r=R_1}. \tag{6$''$}$$

On the wall ($r = R_1$), we prescribe condition (6″) for an ideal liquid metal–solid wall thermal contact. This condition is more rigid than that of the convective heat exchange, which will lead to a less favorable stress distribution than the actual one. This gives a safety factor.

Boundary-value problems (1)–(6), (6′), and (6″) describe the coupled problem of heat and mass transfer.

Replacing boundary-value problems (1), (4), (2), and (3) by the variational problems equivalent to them with a search for a functional minimum at a fixed instant of time, we apply the finite-element method, the Ritz procedure, and the Crank–Nicholson difference scheme. As a result, we obtain the system of difference equations [2]

$$A_1 T_{i,j} - A_2 T_{i,j-1} - A_3 T_{i+1,j} - A_4 T_{i-1,j} - A_5 T_{i,j+1} = 0,$$

$$F_1 c_{i,j}^{k+1} + F_2 c_{i,j-1}^{k+1} + F_3 c_{i+1,j}^{k+1} + F_4 c_{i-1,j}^{k+1} + F_5 c_{i,j+1}^{k+1} + F_6 c_{i+1,j-1}^{k+1} + F_7 c_{i-1,j+1}^{k+1} =$$

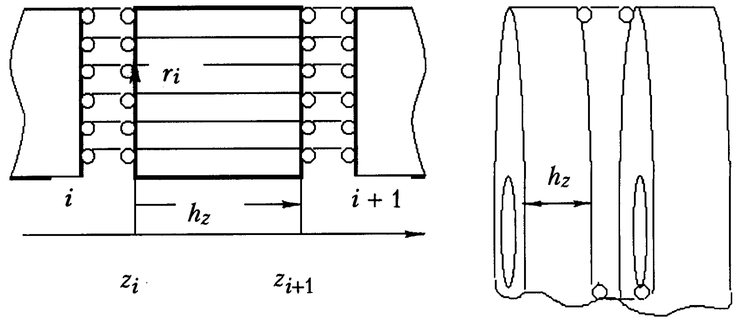

Fig. 2. Scheme of subdivision of the cylinder into superelements.

$$= H_1 c_{i,j}^k + H_2 c_{i,j-1}^k + H_3 c_{i+1,j}^k + H_4 c_{i-1,j}^k + H_5 c_{i,j+1}^k + H_6 c_{i+1,j-1}^k + H_7 c_{i-1,j+1}^k \,, \tag{7}$$

where $i = 2, ..., N$; $j = 2, ..., M$; $k = 0, ..., P$, P is the number of time steps; $c_{i,j}^k$ is the carbon concentration at the node (i, j) at the instant of time t_k; $T_{i,j} = T(z_i, r_j)$; $A_1, A_2, ..., A_5, F_1, F_2, ..., F_7, H_1, H_2, ..., H_7$ are the known coefficients dependent on the coordinates z_i and r_j of the nodes and on the average values of the coefficients of thermal conductivity and diffusion in the elements; $(N + 1)\cdot(M + 1)$ is the number of nodes, $c_{1,j}^0 = 0$, $c_{N+1,j}^0 = 0$, $c_{i,M+1}^0 = 0$, and $c_{i,1}^k = c(t_k, z_i)$.

The difference analogs of Eq. (5) and conditions (6)–(6″) with the use of the "upwind" approximation of a derivative are as follows [3]:

$$c_p \rho V_j \frac{\theta_{i,j} - \theta_{i-1,j}}{h_z} = \frac{\lambda_{\text{sod}}}{h_r^2 r_j} \left[r_{j+1/2} \left(\theta_{i,j+1} - \theta_{i,j} \right) - r_{j-1/2} \left(\theta_{i,j} - \theta_{i,j-1} \right) \right], \quad j = 2, ..., S\,; \quad i = 2, ..., N\,;$$

$$c_p \rho V_1 \frac{h_r^2}{4} \left(\theta_{i,1} - \theta_{i-1,1} \right) = \lambda_{\text{sod}} \left(\theta_{i,2} - \theta_{i,1} \right) h_z \,, \quad i = 2, ..., N \,, \tag{8}$$

where $V_j = V_z(r_j) = V_0 \left(1 - \frac{r_j^2}{R_1^2} \right)$; $r_{j\pm1/2} = r_j = \pm \frac{h_r}{2}$; $\theta_{1,j} = T_{\text{inl}}$ $(j = 1, ..., S)$; $V_0 = \frac{2G}{\rho \pi R_1^2}$.

The coupled heat and mass transfer problem (7)–(8) is solved following the scheme described in [4]. Here, in order that the temperature fields in both the wall and the coolant and the concentration of carbon in the wall be determined, at each time step we use the iteration method. The thermoelasticity problem must also be solved at each time step of this kind. To do this, we apply the method of superelements. The cylinder is subdivided into N superelements, i.e., "disks" of the same length h_z connected with each other by longitudinal couplings (Fig. 2).

The transverse shear of the "disks" is allowed, but the equilibrium condition

$$\tau_{rz} = -\frac{1}{r} \int_{R_1}^{r} \frac{\partial \sigma_z}{\partial z} r dr \tag{9}$$

is fulfilled by analogy with the transverse bending of a beam [5].

Solving the thermoelasticity problem, we assume that within the limits of each superelement the axial strain is constant, but it changes from "disk" to "disk."

The distribution laws for the Young modulus E, the linear expansion coefficient α, and the Poisson ratio μ are taken to be dependent on the temperature T and the concentration of carbon c in the form $E = E_0(T)(1 - \beta(T)\, c)$, $\alpha = \alpha_0(T)(1 - \gamma(T)\, c)$, and $\mu = \mu_0(T)(1 - \delta\,(T)\, c)$.

To solve the thermoelasticity problem for each "disk" we use the finite-element method. The distribution of the temperature and the carbon concentration over the "disk" radius is determined as the arithmetical mean over the

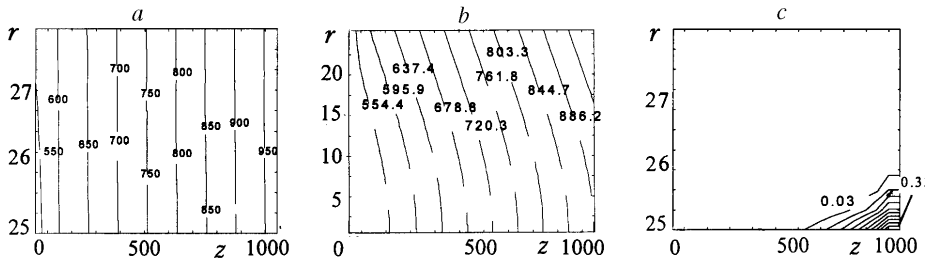

Fig. 3. Isotherms in the tube (a) and in the sodium flow (K) (b) and the lines of equal values of the concentration of carbon (%) (c) after 2000 h. r, z, mm.

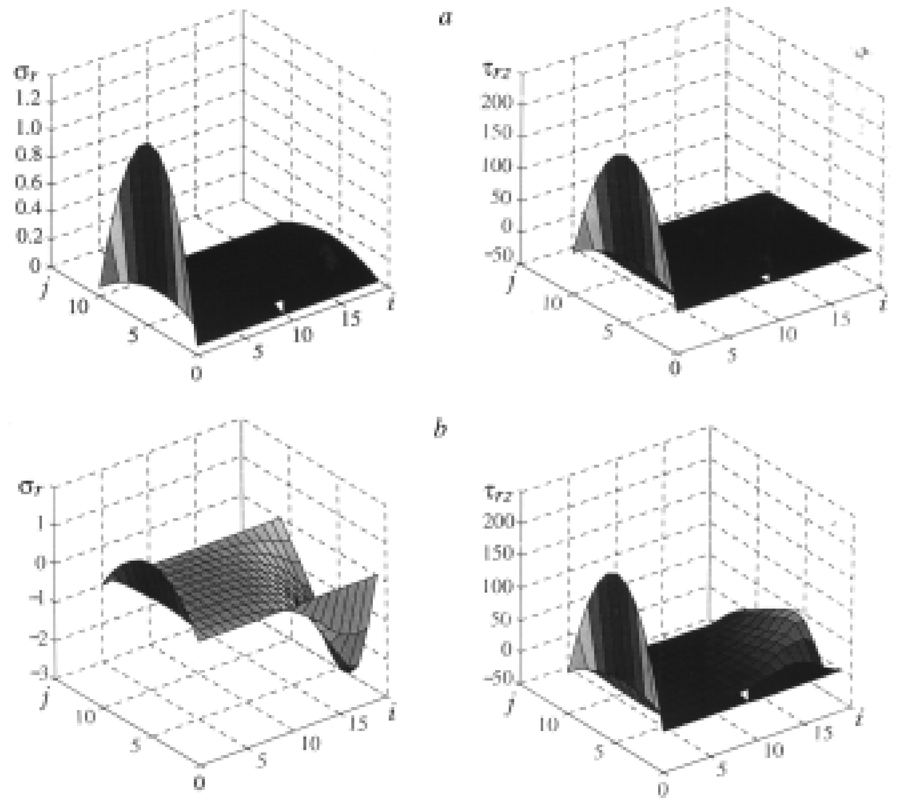

Fig. 4. Distribution of stresses (MPa) in the tube after 2000 h without account for the swelling effect (a) and with account for this effect (b) (i, number of the superelement, j, number of the radial node). σ_r, τ_{rz}, MPa.

"disk" thickness for each ring $T_j = (T_{i,j} + T_{i+1,j})/2$ and $c_j = (c_{i,j} + c_{i+1,j})/2$, where $i = 1, ..., N$; $j = 1, ..., M$. We subdivide the "disk" over the radius into M layers and apply the procedure of [6] to determine the stresses.

The circumferential stresses in the layers will be determined by summing their values, specified by contact pressure and found by virtue of the thin-walled nature of the layers according to the theory of zero-moment shells, with the stresses from the temperature drop over the cylinder-wall thickness and the stresses caused by the swelling of the crystal lattice when carbon penetrates into it. To determine the axial stresses σ_z we take as a basis the hypothesis for the plane strain $\varepsilon_z = \xi = \text{const}$.

Having obtained the radial, circumferential, and axial stresses $\sigma_r(i, j)$, $\sigma_\theta(i, j)$, and $\sigma_z(i, j)$ ($i = 1, ..., N$; $j = 1, ..., M + 1$), from formula (9) we determine the tangential stresses. The numerical differentiation is here carried out using the Lagrange formula with five nodes, while the integration is carried out using the Simpson formula.

We consider the example of calculation performed for a cylinder with geometric dimensions $R_1 = 25$ mm, $R_2 = 28$ mm, and $L = 1000$ mm; the temperature on the cylinder surfaces varies by the linear law:

TABLE 1. Values of the Mechanical and Thermophysical Characteristics of the Material

T, K	Characteristics	c					
		0	0.2	0.4	0.6	0.8	1
873	λ_{st}	0.330	0.237	0.207	0.187	0.177	0.174
	E	127	126	125	124	122	121
	α	1.59	1.59	1.59	1.58	1.58	1.58
973	λ_{st}	0.343	0.247	0.217	0.197	0.187	0.186
	E	117	116	115	114	113	111
	α	1.65	1.64	1.64	1.64	1.63	1.63

$T_2(z) = \tilde{T}_{21} + \dfrac{\tilde{T}_{22} - \tilde{T}_{21}}{L} z$ on the outer surface ($\tilde{T}_{21} = 563$ K and $\tilde{T}_{22} = 973$ K), $T_3(r) = \tilde{T}_{31} + \dfrac{\tilde{T}_{32} - \tilde{T}_{31}}{R_2 - R_2}(r - R_1)$ on the left end, and $T_4(r) = \tilde{T}_{41} + \dfrac{\tilde{T}_{42} - \tilde{T}_{41}}{R_2 - R_2}(r - R_1)$ on the right end ($\tilde{T}_{31} = \tilde{T}_{11} = T_{inl} = 513$ K, $\tilde{T}_{32} = \tilde{T}_{21}$, and $\tilde{T}_{42} = \tilde{T}_{22}$); 1Kh18N10T steel is used as the material. The following data obtained from [1, 7] were taken in the calculations: $D = D_0 \exp(-k_0/T)$, where $k_0 = 11{,}150$ K and $D_0 = 7.4443\ \text{mm}^2/\text{h}$. The dependence of the thermal-conductivity coefficient $\lambda_{st}(c,T)$ [J/(mm·h·K)] on the concentration of carbon c and the temperature T and the values of the Young modulus E (GPa) and of the linear expansion coefficient $\alpha \cdot 10^5$ (1/deg) (which is taken hypothetically) are presented in Table 1. The Poisson ratio is $\mu = 0.3$, $B = 2340.87$, $k(T) = 1 - 0.00225(T - 873\ \text{K})$, $n = 0.52$. The swelling parameters λ_i ($i = 1, \ldots, 5$) are the same as for 45Kh steel and have the values $\lambda_1 = 0.0158$, $\lambda_2 = -0.0855$, $\lambda_3 = 0.2143$, $\lambda_4 = -0.2422$, and $\lambda_5 = 0.1024$ [8]. The thermophysical characteristics of liquid sodium are as follows [9]: $c_p = 1300$ J/(kg·deg), $\lambda_{sod} = 80$ W/(m·deg), and $\rho = 900\ \text{kg/m}^3$. The mass flow rate of liquid sodium is $G = 0.1$ kg/sec. The number of subdivision elements was taken to be equal to $N = 20$, $M = 10$, and $S = 50$. The time step was to 500 h. These parameters of subdivision turned out to be sufficient to attain the required accuracy.

The calculation results are given in Figs. 3 and 4. In all the cases considered here we used a thermosensitive material whose properties were dependent on the concentration of carbon.

The analysis of the results obtained has shown that account for the swelling effect leads not only to a quantitative change in the stressed state but also to qualitative changes, which demonstrates the need for taking into account the indicated factor.

NOTATION

R_1 and R_2, inner and outer radii of the tube, respectively; L, tube length; r and z, cylindrical coordinates; t, time; T and θ, temperatures of the tube and liquid sodium; T_{inl}, temperature of liquid sodium at the inlet to the tube; G, mass flow rate of liquid sodium; V_r and V_z, transverse and longitudinal components of the velocity of the sodium flow; V_0, velocity of the sodium flow on the tube axis; Pe, Péclet thermal number; c_p, specific heat of liquid sodium at constant pressure; p, liquid-sodium density; c, relative concentration of carbon; D, diffusion coefficient; Q, activation energy; R, gas constant; D_0, constant for a given material characterizing the diffusion coefficient; B, constant characterizing the change in the carbon concentration; λ_{st} and λ_{sod}, thermal-conductivity coefficients of the steel and sodium; T_2, T_3, and T_4, temperatures on the outer surface and on the ends of the cylinder; r_j and z_i, cylindrical coordinates of the grid nodes; V_j, nodal values for the velocity of the sodium flow; $c_{i,j}$, $T_{i,j}$, and $\theta_{i,j}$, nodal values of the carbon concentration and of the temperature of the steel and liquid sodium; σ_r, σ_θ, σ_z, and τ_{rz}, radial, circumferential, axial, and tangential stresses.

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